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Calculable Dynamical Supersymmetry Breaking on Deformed Moduli Spaces

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Abstract: We consider models of dynamical supersymmetry breaking in which the extremization of a tree-level superpotential conflicts with a quantum constraint. We show that in such models the low-energy effective theory near the origin of moduli space is an O'Raifeartaigh model, and the sign of the mass-squared for the pseudo-flat direction at the origin is calculable. We analyze vector-like models with gauge groups $SU(N)$ and $Sp(2N)$ with and without global symmetries. In all cases there is a stable minimum at the origin with an unbroken $U(1)_R$ symmetry.

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A very simple mechanism for breaking supersymmetry (SUSY) dynamically was introduced in [1, 2]. The starting point is a model with a moduli space that is deformed by quantum effects [3]. If we denote the holomorphic gauge-invariants that parameterize the moduli space by M, a deformation can be written

$$C(M) = \Lambda^n \,, \tag{1}$$

where $C(M)$ is a homogeneous polynomial that vanishes in the classical limit $\Lambda \to 0$. The idea of [1, 2] is to add a tree-level superpotential to such a theory chosen so that extremizing the tree-level superpotential (expressed in terms of the fields M) conflicts with the quantum constraint. (The simplest possibility is to arrange the tree-level superpotential so that F-flatness forces all of the M's to vanish.) Since there is no solution of the F-flatness conditions consistent with the quantum constraint, SUSY is broken. The justification for this simple argument comes from non-perturbative SUSY non-renormalization theorems [4, 3], and is discussed in [1, 2]. A particularly interesting feature of this mechanism is that it can occur in non-chiral theories.

The simplest theory of this kind has gauge group $SU(2)$ with 4 fundamental fields Q, 6 singlets S, and tree-level superpotential

$$W = \lambda S_{jk} Q^j Q^k \,, \tag{2}$$

where $j, k = 1, \ldots, 4$ are 'flavor' indices, and $S_{jk} = -S_{kj}$. This model has an anomaly-free $U(1)_R$ symmetry with $R(Q) = 0$, $R(S) = +2$. The moduli space can be parameterized by the holomorphic gauge-invariants

$$M^{jk} = Q^j Q^k = -M^{kj} \,. \tag{3}$$

Classically, these satisfy the constraint $\mathrm{Pf}(M) = 0$, but this is modified by quantum effects to [3]

$$\mathrm{Pf}(M) = \Lambda^4 \,. \tag{4}$$

The equations $\partial W / \partial S_{jk} = 0$ set $M^{jk} = 0$ for all j, k; this conflicts with the quantum constraint, so SUSY is broken.

There is one linear combination of the fields S whose VEV is undetermined in the approximation where the Kähler potential for S is quadratic. This is the tree-level flat direction that exists in any O'Raifeartaigh model. Analysis of the theory for $\langle S \rangle \gg \Lambda$ [5] shows that there is no minimum for large values of S, and so the global minimum is either at $\langle S \rangle = 0$ or $\langle S \rangle \sim \Lambda / \lambda$.[1] In the literature it is stated that the sign of the mass-squared for S near the origin $S = 0$ depends on unknown strong interaction terms in the effective Kähler potential, and is therefore uncalculable. In this paper we analyze the theory near $S = 0$ and show that the low-energy theory

[1] We are ignoring factors of 4π for most of this discussion. We will indicate how these enter when we state our final results.

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is precisely an O'Raifeartaigh model. The Kähler effects that determine the S flat direction are dominated by loops of light fields, and are therefore calculable for perturbative values of λ. We find that there is a local minimum at the origin where the anomaly-free $U(1)_R$ symmetry is unbroken.

Near $S = 0$ and for energies below Λ, the effective theory can be written in terms of the composite 'meson' fields M and the singlets S. The analysis is simplified by making use of the (Lie algebra) isomorphism between $SU(4)$ and $SO(6)$. In $SO(6)$ language, we write the mesons as M_a, $a = 1, \ldots, 6$ with constraint

$$M_a M_a = \Lambda^2 \,, \tag{5}$$

where we have rescaled by factors of Λ so that M_a has mass dimension $+1$. The superpotential is then

$$W_{\text{eff}} = \lambda \Lambda S_a M_a \,. \tag{6}$$

We can solve the constraint to write $M_1 = \pm(\Lambda^2 - M'^2)^{1/2}$, where $M'_a = M_a$ for $a = 2, \ldots, 6$. This gives a superpotential

$$W_{\text{eff}} = \lambda \left[S_1 (\Lambda^2 - M'^2)^{1/2} + \Lambda S' M' \right] , \tag{7}$$

where $S'_a = S_a$ for $a = 2, \ldots, 6$.

Eq. (7) shows that this model breaks SUSY via the O'Raifeartaigh mechanism: the conditions $\partial W_{\text{eff}} / \partial S_1 = 0$ and $\partial W_{\text{eff}} / \partial S'_a = 0$ cannot be simultaneously satisfied. Minimizing the potential using the approximation that the Kähler potential is quadratic gives

$$\operatorname{Im} M'_a = 0 \,, \qquad S'_a = \frac{S_1 M'_a}{(\Lambda^2 - M'^2)^{1/2}} \,. \tag{8}$$

Note that $\operatorname{Re} M'_a$ and S_1 are undetermined, and hence the corresponding fluctuations are massless at this level. The fields $\operatorname{Re} M'_a$ correspond to the 5 Nambu–Goldstone bosons resulting from the spontaneous breaking $SO(6) \to SO(5)$ (or $SU(4) \to Sp(4)$), and are therefore exactly massless.[2] The tree-level potential therefore precludes the other possible symmetry breaking pattern $SO(6) \to SO(4)$ (or $SU(4) \to SU(2) \times SU(2)$).

The field S_1 parameterizes the tree-level flat direction that is present in all O'Raifeartaigh models. Vacua with different values of S_1 are not physically equivalent (*e.g.* the $U(1)_R$ symmetry is broken if $S_1 \neq 0$), and this degeneracy is lifted when we include non-minimal terms in the effective Kähler potential and loop corrections in the effective theory. There are non-calculable terms in the effective Kähler potential from the strong interactions at the scale Λ:

$$K_{\text{eff}} = S^\dagger S + M'^\dagger M' + \Lambda^2 f \left(\frac{\lambda S}{\Lambda}, \frac{M'}{\Lambda} \right) . \tag{9}$$

[2]The fields $\operatorname{Re} M'_a$ are stereographic coordinates for the compact space $SO(6)/SO(5)$.

We are interested in vacua satisfying eq. (8), where

$$\frac{\partial W_{\rm eff}}{\partial M'_a} = S'_a - \frac{S_1 M'_a}{(\Lambda^2 - M'^2)^{1/2}} = 0 \,. \tag{10}$$

Therefore, only the 'SS' entries of the inverse Kähler metric contribute to the potential for S_1. The leading contribution from the uncalculable Kähler potential that lifts the S_1 flat direction therefore comes from a quartic term of order $|\lambda|^4 (S^\dagger S)^2/\Lambda^2$, which gives a contribution to the potential of order $|\lambda|^6 \Lambda^2 |S_1|^2$.

However, there are also corrections coming from loops of light particles in the effective theory. If we consider vacua where $\langle S_1 \rangle \ll \Lambda/\lambda$, then the light massive particles have mass of order $m \sim \lambda \langle S_1 \rangle \ll \Lambda$. The effective potential obtained by integrating out these massive particles at one loop is of order $m^4 \sim |\lambda|^4 |S_1|^2$, which is larger than the contribution from the uncalculable Kähler terms. Therefore, the question of the stability of the vacuum $S_1 = 0$ can be answered in perturbation theory in the effective theory. Putting in the factors of 4π in these estimates both from the weak loops and the strong interactions [6], we find that the calculable contributions to the Kähler potential are larger than the non-calculable ones by $\mathcal{O}(16\pi^2/\lambda^2)$, so calculability breaks down only when λ has non-perturbative strength.

The 1-loop contribution to the effective potential is

$$V^{(1)}_{\rm 1PI} = \frac{1}{64\pi^2} \, {\rm str} \left(\mathcal{M}^4 \ln \frac{\mathcal{M}^2}{\mu^2} \right), \tag{11}$$

where $\mathcal{M}^2$ is the mass-squared matrix of the scalar and fermion fields evaluated as a function of background scalar field VEV's. Because we are interested only in the potential of the light fields, we can set the VEV's of the massive scalar fields ${\rm Im}\, M'$ and S' equal to their tree-level values.[3] Note that there can be no mixing between the light fields ${\rm Re}\, M'_a$ and S_1 because of the unbroken $SO(5)$ symmetry. We can therefore fix ${\rm Re}\, M' = 0$ as well. Evaluating eq. (11) by brute force gives

$$V^{(1)}_{\rm 1PI} = + \frac{5|\lambda|^4 \Lambda^2}{16\pi^2} (2\ln 2 - 1) |S_1|^2 + \mathcal{O}(S_1^4) \,. \tag{12}$$

We see that the mass of S_1 is positive at $S_1 = 0$.

The approximations made above break down when $\lambda S \sim \Lambda$; in this case, there are states in the effective theory with mass of order Λ, the mass scale of strong resonances. In this case there is no clean separation between 'low-energy' and 'high-energy' physics. We therefore cannot exclude the possibility that there is a global minimum with $\langle S \rangle \sim \Lambda/\lambda$.

[3]Strictly speaking, we should do a matching calculation to integrate out the massive fields and write the effective theory for the light fields. However, the resulting effective theory has no dimensionless couplings, and so there are no large logarithms to worry about. We can therefore do a straightforward calculation in the full theory without missing any large effects.

In the remainder of the paper, we consider some generalizations of the model considered above. We first consider breaking the $SO(6)$ symmetry explicitly by choosing the Yukawa couplings in the tree-level superpotential to break the flavor symmetry in an arbitrary way:

$$W_{\text{eff}} = \sum_a \lambda_a \Lambda S_a M_a \,. \tag{13}$$

We can choose all the λ_a to be real by rephasing the S_a. For generic λ_a, there are no massless Nambu–Goldstone bosons, and there is an additional contribution to the S_1 mass of the same order as eq. (12). We want to know whether this can destabilize the vacuum at $S_1 = 0$. Assume without loss of generality that λ_1 is the smallest Yukawa coupling. We then find that the tree-level vacuum is given by eq. (8), and a direct calculation gives

$$V_{\text{1PI}}^{(1)} = \frac{1}{64\pi^2} \sum_{a=2}^{6} f(\lambda_a^2/\lambda_1^2)|S_1|^2 + \mathcal{O}(S_1^4) \,, \tag{14}$$

where

$$f(x) = 2(x+1)^2 \ln(x+1) - 2(x-1)^2 \ln(x-1) - 8x \ln(x) - 4x \,. \tag{15}$$

$f(x) \geq 0$ for all $x \geq 1$, so the S_1 mass-squared is positive for arbitrary values of λ_a.

We now turn to another model with gauge group $SU(N)$ and N flavors of quarks Q^j, $\bar{Q}_{\bar{k}}$. In addition, the model contains N^2+2 singlets $S^{\bar{k}}{}_j$, T, and $\bar{T}$, with tree-level superpotential

$$W = \lambda S^{\bar{k}}{}_j Q^j \bar{Q}_{\bar{k}} + \kappa \left[T \det(Q) + \bar{T} \det(\bar{Q}) \right]. \tag{16}$$

Here, λ is dimensionless, and κ has mass dimension $-(N-2)$. In order for this model to be consistent as an effective theory, we require

$$\kappa \ll \frac{1}{\Lambda^{N-2}} \,, \tag{17}$$

so that the higher-dimension operator in the superpotential is weak above the scale where the $SU(N)$ dynamics becomes strong. The non-perturbative dynamics generates a deformed moduli space with

$$\det(M) - B\bar{B} = \Lambda^{2N} \,, \tag{18}$$

where $M^j{}_{\bar{k}} \equiv Q^j \bar{Q}_{\bar{k}}$ are composite 'meson' fields, and $B \equiv \det(Q)$, $\bar{B} \equiv \det(\bar{Q})$ are composite 'baryon' fields. The quantum constraint conflicts with the F-flat constraints from the superpotential, so this model breaks SUSY by the mechanism of [1, 2].

It is convenient to introduce the fields

$$B_\pm \equiv \frac{1}{\sqrt{2}}(B \pm \bar{B}) \,, \qquad T_\pm \equiv \frac{1}{\sqrt{2}}(T \pm \bar{T}) \,, \tag{19}$$

so that the constraint reads

$$\det(M) - B_+^2 + B_-^2 = \Lambda^{2N} \,. \tag{20}$$

We then write the effective superpotential as

$$W_{\text{eff}} = \lambda S^{\bar{k}}{}_j M^j{}_{\bar{k}} + \kappa \left[T_+ B_+ + T_- (\Lambda^{2N} - \det(M) + B_+^2)^{1/2} \right], \tag{21}$$

where we have solved the constraint for B_-. The first term gives a mass of order $\lambda\Lambda$ to the fields S and M. The remaining terms are highly suppressed (assuming $\kappa \ll \lambda/\Lambda^{N-2}$), and can be treated as a small perturbation at the scale $\lambda\Lambda$. Integrating out the massive fields S and M, we obtain an effective theory with fields $T_\pm$ and B_+ and superpotential

$$W_{\text{eff}} = \kappa \left[T_+ B_+ + T_- (\Lambda^{2N} + B_+^2)^{1/2} \right]. \tag{22}$$

This has the same form as the superpotential in the $SU(2)$ model, but there is only one composite field. If we rescale B_+ by powers of Λ to give it mass dimension $+1$, we find that the dimensionless expansion parameter in this theory is $\kappa\Lambda^{N-2} \ll 1$, and the previous analysis tells us that there is a local minimum at $T_\pm = 0$, $B_+ = 0$.

The final model we consider has gauge group $Sp(2N)$ with $2N+2$ fundamentals Q^j, $j = 1, \ldots, 2N+2$. We also add $N(2N-1)$ singlets $S_{jk} = -S_{kj}$, and a tree-level superpotential

$$W = \lambda S_{jk} Q^j Q^k \,. \tag{23}$$

The quantum constraint is

$$\text{Pf}(M) = \Lambda^{2N} \,, \tag{24}$$

where $M^{jk} \equiv Q^j Q^k = -M^{kj}$. To solve the constraint, we introduce the notation

$$[A_1 \cdots A_N] \equiv \frac{1}{2^N N!} \epsilon_{j_1 \cdots j_{2N}} A_1^{j_1 j_2} \cdots A_N^{j_{2N-1} j_{2N}} \,, \tag{25}$$

so that $[A \cdots A] = \text{Pf}(A)$. We then write

$$M^{jk} = M_0 J^{jk} + M'^{jk} \,, \tag{26}$$

where J^{jk} is the $Sp(2N)$ metric (normalized so that $\text{Pf}(J) = 1$), and M'^{jk} satisfies $\text{tr}(JM') = 0$.[4] We expand around $M = \Lambda J$ and treat M' as a perturbation. The constraint can then be written

$$\begin{aligned} \Lambda^{2N} &= [(M_0 J + M') \cdots (M_0 J + M')] \\ &= M_0^N + N M_0^{N-1} [M' J \cdots J] + \frac{N(N-1)}{2} M_0^{N-2} [M' M' J \cdots J] + \mathcal{O}(M'^3) \,. \end{aligned} \tag{27}$$

[4]Our index conventions are $J^{jk} = J_{jk}$, so that the matrix notation is unambiguous.

Using

$$[M'J\cdots J] = -\frac{1}{2N}\,\mathrm{tr}(M'J) = 0\,, \tag{28}$$

$$[M'M'J\cdots J] = -\frac{1}{2N(N-1)}\,\mathrm{tr}(M'JM'J)\,, \tag{29}$$

we can write

$$M_0^N = \Lambda^{2N} + \tfrac{1}{4}\Lambda^{2N-4}\,\mathrm{tr}(M'JM'J) + \mathcal{O}(M'^3)\,. \tag{30}$$

In terms of the canonically normalized fields S_0 and S' defined by

$$S_{jk} = \frac{1}{\sqrt{2N}}S_0 J_{jk} + (S')_{jk}\,, \qquad \mathrm{tr}(S'J) = 0\,, \tag{31}$$

the effective superpotential can be written

$$W_{\text{eff}} = \lambda\left\{\sqrt{2N}S_0\left[\Lambda^2 + \frac{1}{4N}\,\mathrm{tr}(M'JM'J) + \mathcal{O}(M'^3)\right] + \Lambda\,\mathrm{tr}(S'M')\right\}. \tag{32}$$

where we have rescaled the field M' by Λ to have mass dimension $+1$.

To simplify this, we use a basis E_a for the vector space of antisymmetric matrices such that $\mathrm{tr}(E_a J E_b J) = \delta_{ab}$. For example, in the basis where

$$J = \begin{pmatrix} \epsilon_2 & & \\ & \ddots & \\ & & \epsilon_2 \end{pmatrix}, \qquad \epsilon_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{33}$$

we can use basis elements of the block form

$$\begin{pmatrix} 0 & & & \\ & \epsilon_2 & & \\ & & 0 & \\ & & & \ddots \end{pmatrix}, \qquad \begin{pmatrix} & & \sigma_j\epsilon_2 & \\ & -(\sigma_j\epsilon_2)^T & & \\ & & & \\ & & & \end{pmatrix}, \tag{34}$$

where $\sigma_0 = 1_2$, $\sigma_{1,2,3} =$ Pauli matrices. To get a basis satisfying $\mathrm{tr}(E_a J) = 0$, we take linear combinations of the block-diagonal basis elements above. Expanding $M' = M'_a E_a$ and $JS'J = S_{1a}E_a$, we obtain

$$W_{\text{eff}} = \lambda\left\{\sqrt{2N}S_0\left[\Lambda^2 + \frac{1}{4N}M'_a M'_a + \mathcal{O}(M'^3)\right] + \Lambda S'_a M_{2a}\right\}. \tag{35}$$

This is again a generalization of the superpotential for the $SU(2)$ model obtained above. In fact, the tree-level potential can be written

$$V = |\lambda|^2\Lambda^2\Bigg[\left(1 + \left|\frac{S_0}{\sqrt{2N}\Lambda}\right|^2\right)M'^\dagger_a M'_a + \frac{1}{2}\,(M'_a M'_a + \text{h.c.}) + \\ + S'^\dagger_a S'_a\left(\frac{S_0}{\sqrt{2N}\Lambda}S'^\dagger_a M'_a + \text{h.c.}\right)\Bigg] + \text{non-renormalizable terms}\,. \tag{36}$$

This is a simple generalization of the potential for the $SU(2)$ model with $a = 1, \ldots, 2N^2 - N - 1$, and S_0 rescaled. The model can be analyzed in exactly the same way, and the conclusion is again that there is a local minimum at $S_0 = 0$.

We can also break the global $SU(2N)$ symmetry explicitly by allowing different Yukawa couplings for different flavors. Keeping terms quadratic in M'_a, the Yukawa couplings reduce to symmetric matrices λ_{ab} that can be diagonalized by an orthogonal transformation acting on the indices a, b. In the diagonal basis, the analysis is the same as in the $SU(2)$ model, and we conclude that the minimum at $S_0 = 0$ is stable for arbitrary flavor breaking.

In conclusion, we have shown that models that break SUSY by the mechanism proposed in [1, 2] reduce to O'Raifeartaigh models near the origin of moduli space, and the sign of the mass-squared of the neutral fields is calculable. We showed that there is a stable minimum at the origin of the pseudo-flat direction that preserves the $U(1)_R$ symmetry in models based on gauge groups $SU(N)$ and $Sp(2N)$, for arbitrary superpotential couplings. It would be interesting to also consider the effect of weakly gauging a subgroup of the global symmetries of this model, but we leave this for future work. Unfortunately the question of whether there is a global (or local) minimum when the pseudo-flat direction has a VEV of order Λ/λ requires knowledge of the Kähler terms induced by the strong interactions, which remain uncalculable.

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